Incremental Transitive Closure for Zonal Abstract Domain

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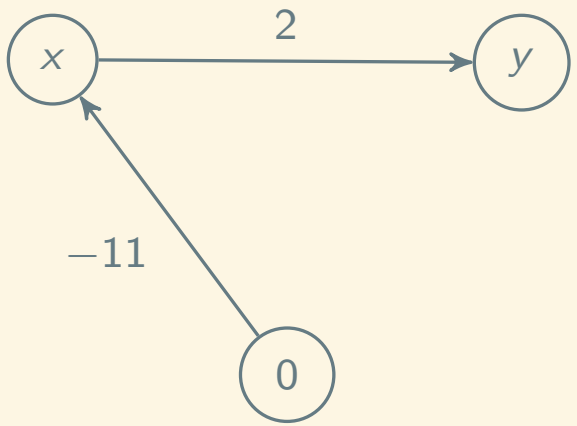
Outline

1. Background
   - Zonal Domain
   - Canonical Representations
2. Incremental Algorithm
   - Example of how the incremental algorithm works
3. Experimental Results
4. Conclusion
5. Future Work

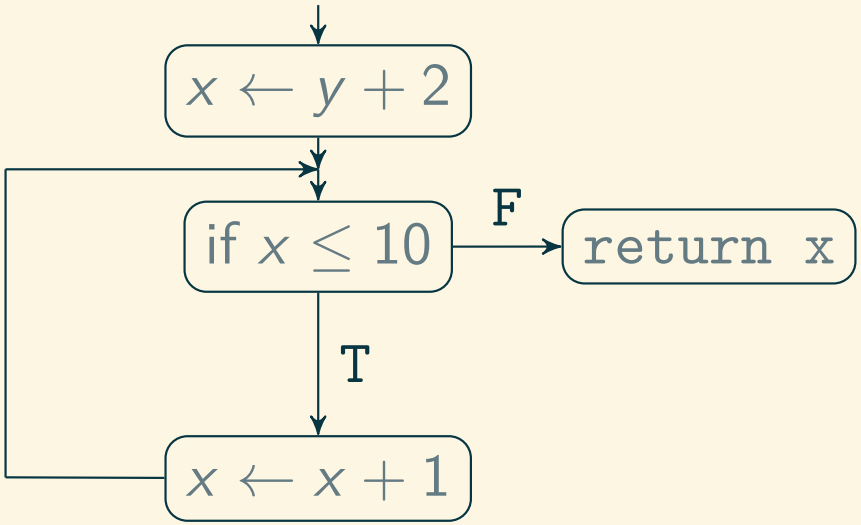
Zonal Domain

$$x - y \leq 2$$
$$0 - x \leq -11$$



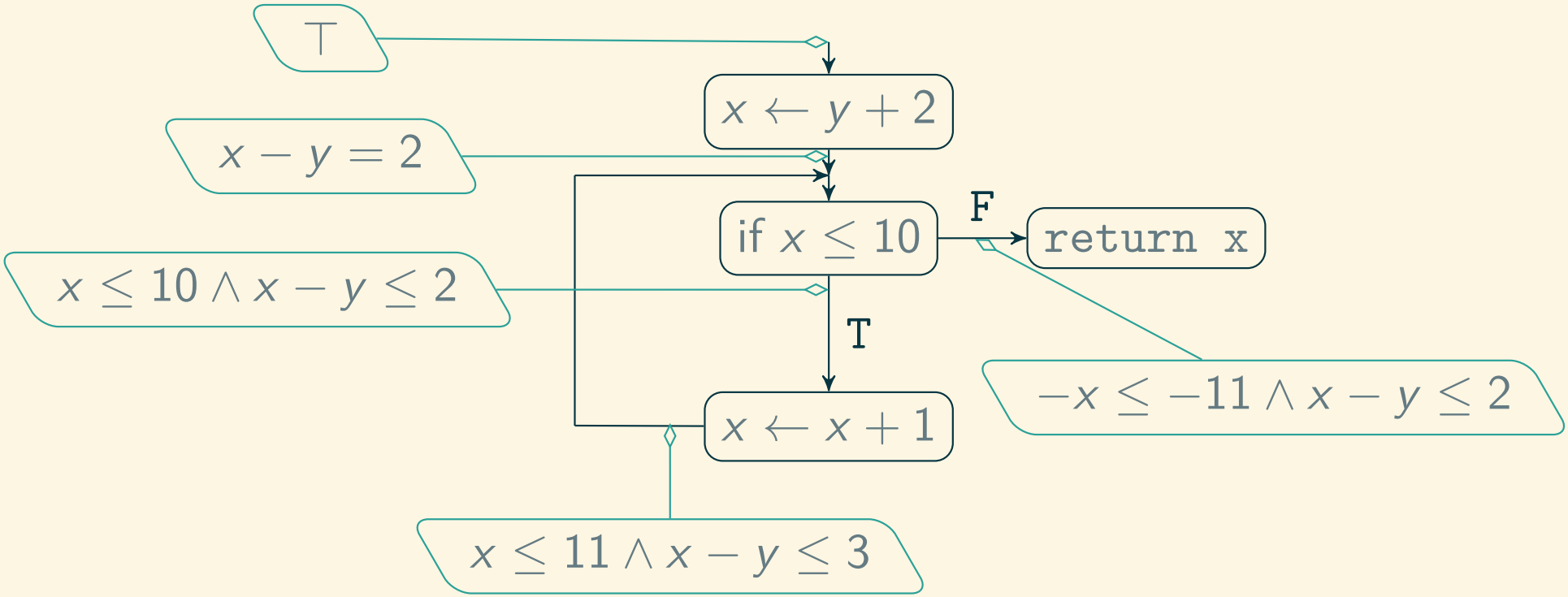
Zonal state representation of data-flow analysis invariant

Example Data Flow Graph

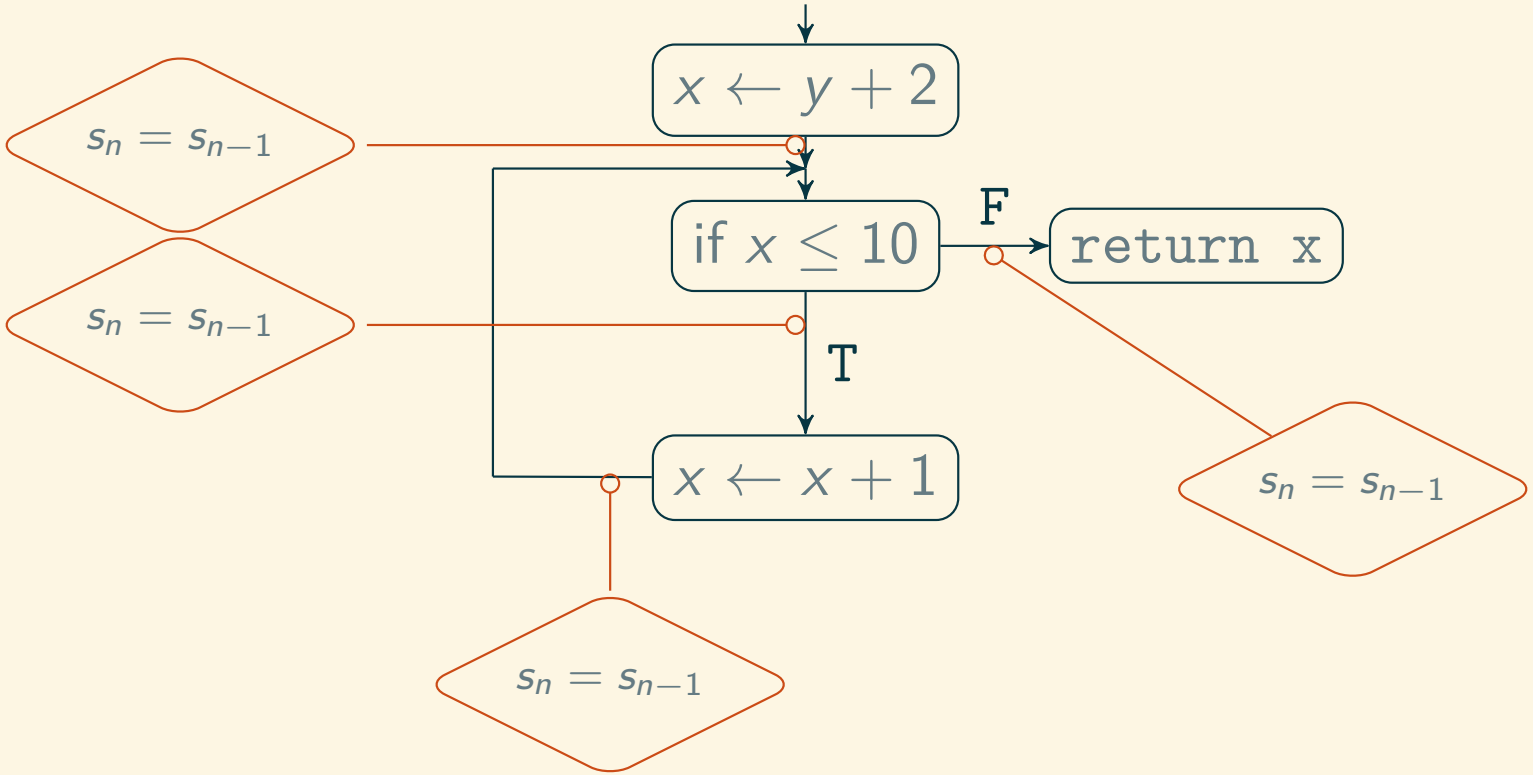


Computed Invariants for Data Flow Graph

Example Zonal Invariants Computed via Data Flow Analysis

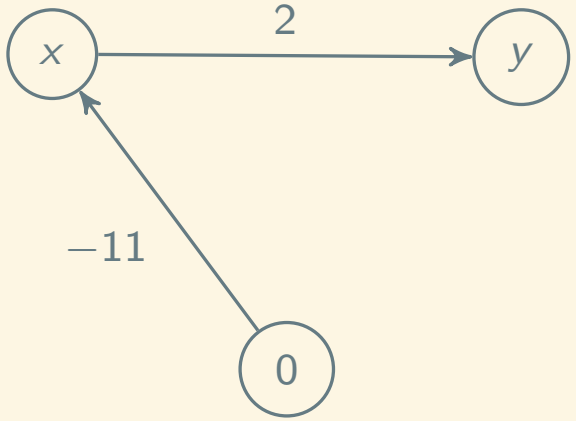


Equality Comparisons During Fixed-Point Computation

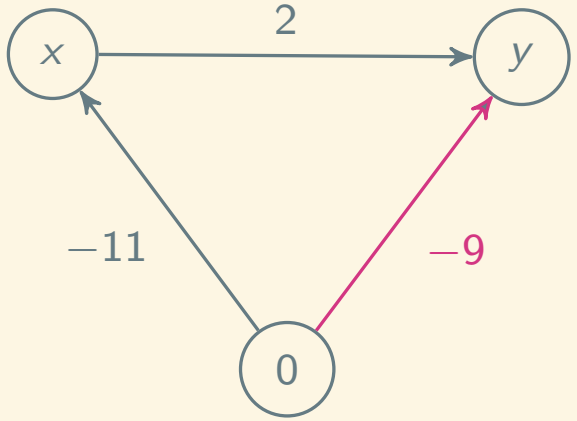


Zonal Domain with Transitive Closure

Canonical Representation



(a) Original state from Analysis



(b) State with inferred relationships

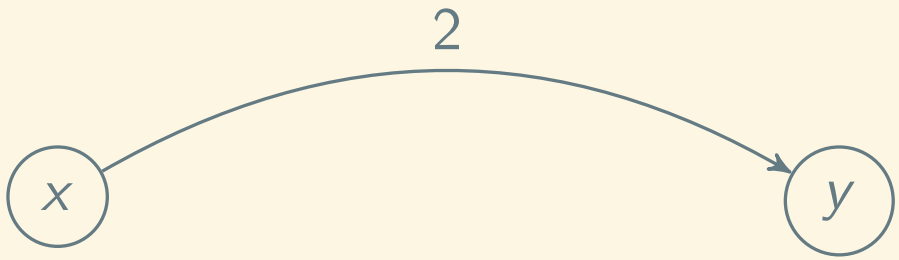
- Transitive closure property: $\forall i, j, k : E_{ij} \leq E_{ik} + E_{kj}$
- Required operation for state comparison
- Provides feasibility information
- Equality comparison is expensive due to computational cost of closure being $\Theta(n^3)$ and due to the frequency of its use.

Observations Leading to Incremental Algorithm

- All-pairs shortest path algorithm dominates analysis time
  - Floyd-Warshall algorithm, for example, is $\Theta(n^3)$
- Fixed-Point computation already propagates fully-closed Zonal state

Incremental Algorithm Example for Adding an Edge

Starting with a fully-closed Zonal State



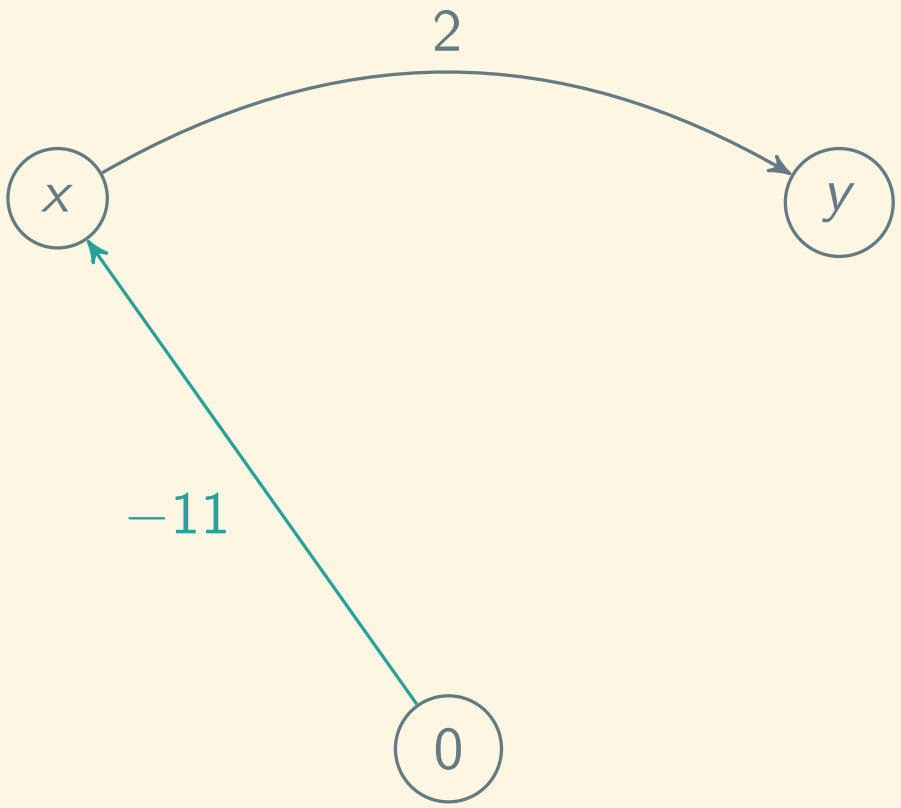
Constraints

$$x - y \leq 2$$

Incremental Algorithm Example for Adding an Edge

Constraints

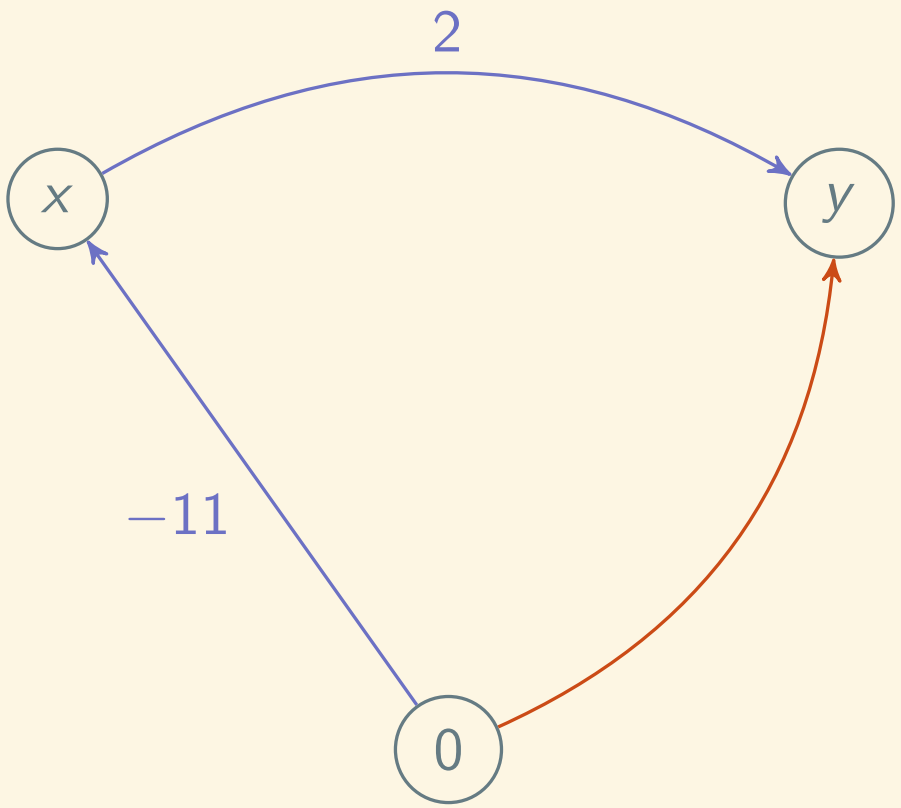
$$x - y \leq 2$$
$$-x \leq -11$$



Incremental Algorithm Example for Adding an Edge

Constraints

$$x - a \leq 2$$
$$-x \leq -11$$
$$-y \leq -9$$

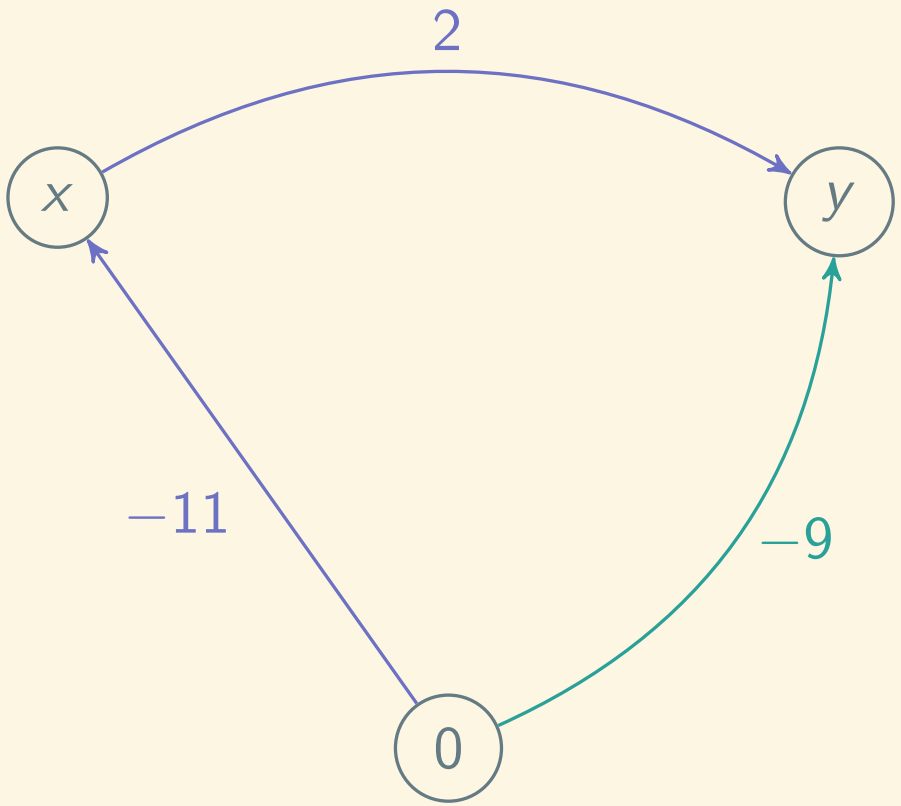


$$0 - y \leq \min(\infty, -11 + 2)$$

Incremental Algorithm Example for Adding an Edge

Constraints

$$x - a \leq 2$$
$$-x \leq -11$$
$$-y \leq -9$$

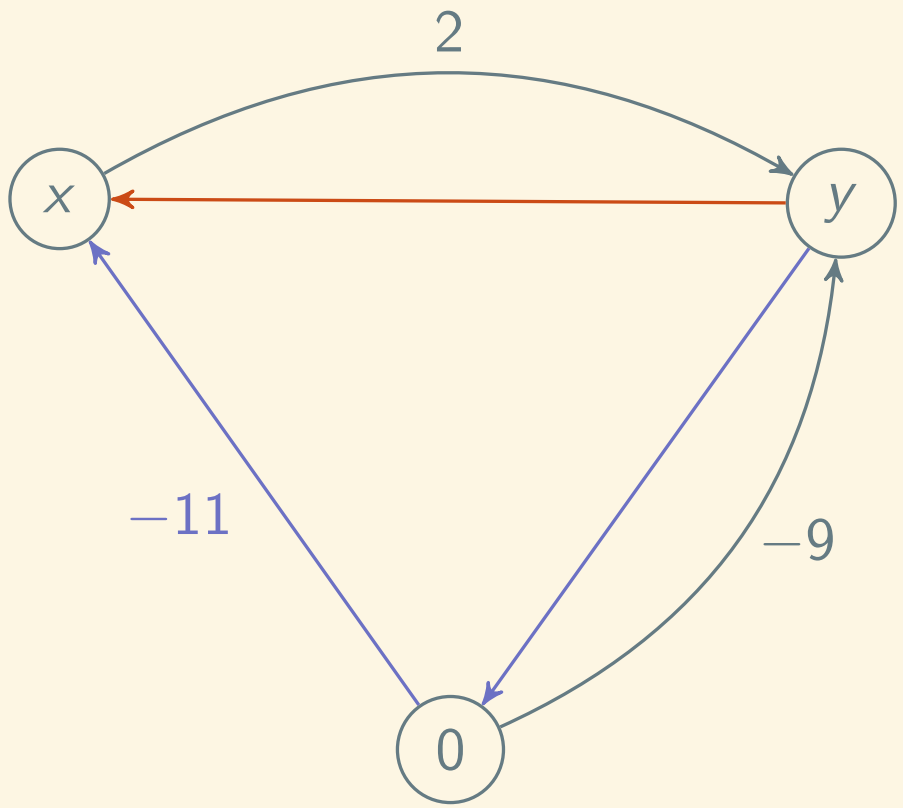


$$-y \leq -9$$

Incremental Algorithm Example for Adding an Edge

Constraints

$$x - y \leq 2$$
$$-x \leq -11$$
$$-y \leq -9$$



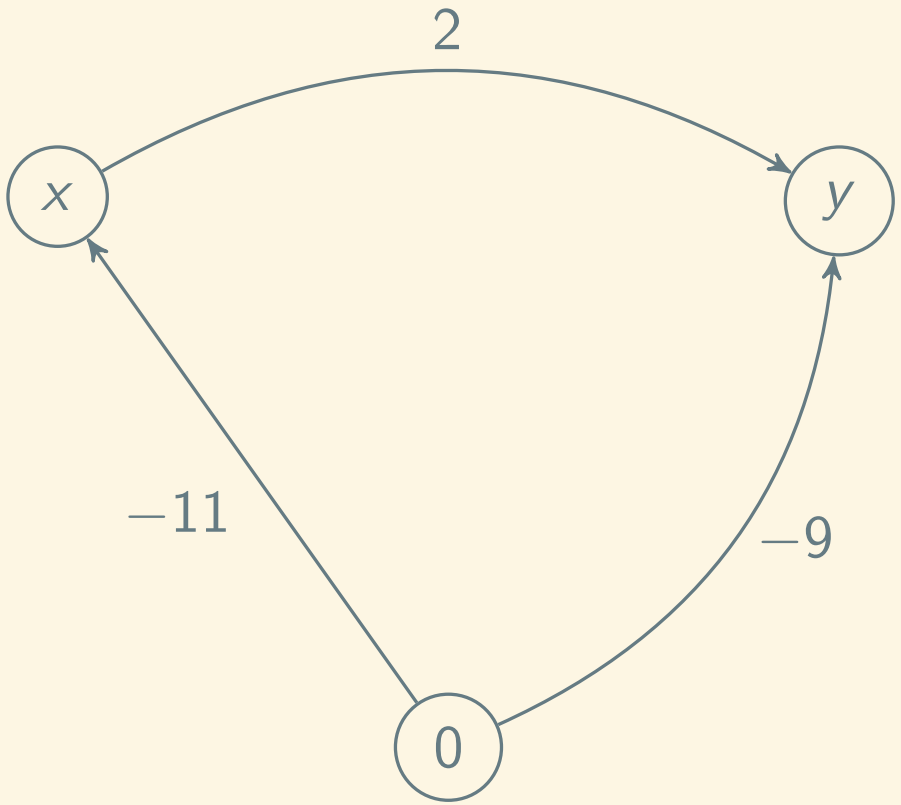
$$y - x \leq \min(\infty, \infty + -11)$$

Incremental Algorithm Example for Adding an Edge

We finish with a closed Zonal state

Constraints

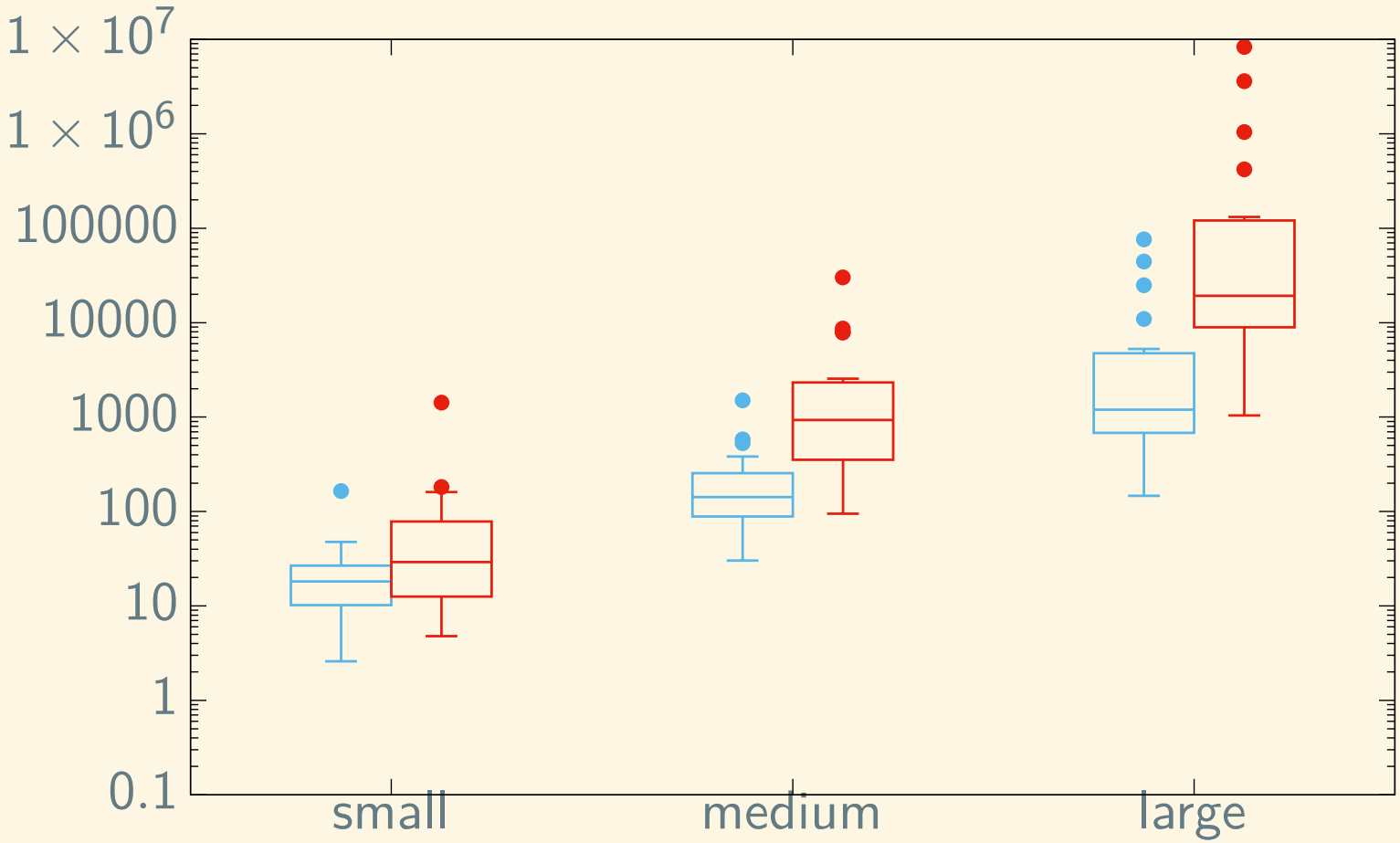
$$x - y \leq 2$$
$$-x \leq -11$$
$$-y \leq -9$$



Experimental Results

- Benchmarks: 63 Java methods
- Methods were divided into three groups based on instructions count
- Average analysis time computed from 3 analysis runs for each sample method

Experimental Results Show Significant Difference in Runtime

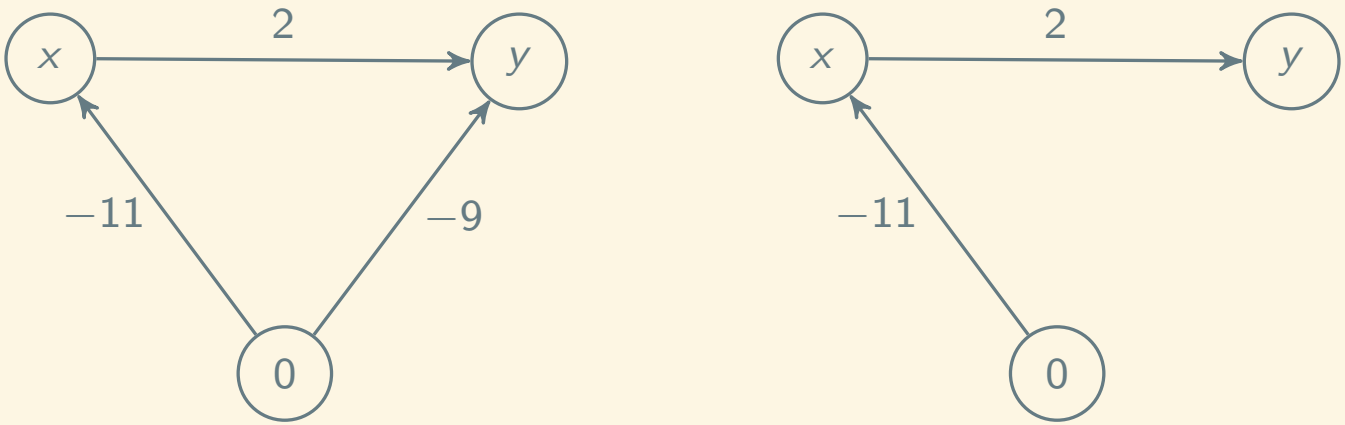


Analysis Timing Results (ms) using Incremental Closure (blue) and Standard Closure (red) Algorithms

Conclusion

- Propagating canonical states presents some advantages in weakly-relational domains such as Zonal Domain.
- Incrementally computing the closure reduces the overall analysis time.
- Some transfer functions can be simplified, e.g., forget.

Future Work



Explore incremental approaches for other canonical representations, e.g., Larsen Reduction.

Thank you

Questions?

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